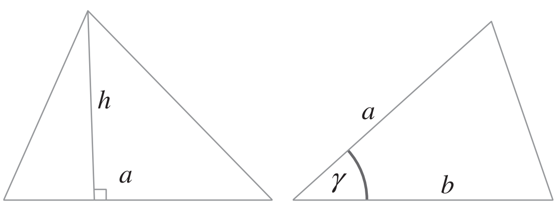
Chapter 5

The Area of a Quadrilateral

Historically, many of the most important geometrical concepts arose from problems that are practical in origin. A good example is the calculation of the area of convex polygons. The area of the triangle as a basic plane polygon is often employed in calculating the area of other polygons. It is most commonly calculated by the formulas $S = \frac{1}{2}ah$ (half the product of the base by the altitude dropped to that base) and $S = \frac{1}{2}ab \cdot \sin\gamma$ (half the product of two sides and the sine of the angle between them), which we have already used a few times.



The area of a triangle in terms of the sides' lengths is found by the *Heron's Formula*, which was first mentioned and proved in the book "Metrica" by the prominent Greek mathematician, engineer, and inventor Heron of Alexandria (c.10–c.70 AD):

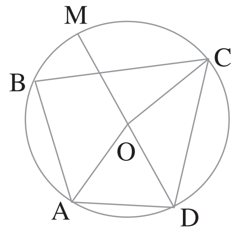
$$S = \sqrt{p(p-a)(p-b)(p-c)},$$

where *a, b,* and *c* are the sides and *p* is the semiperimeter of a triangle.

While the first two formulas are covered in high school, we can't say the same about Heron's Formula. Even though the main focus of this chapter is on the areas of quadrilaterals, Heron's Formula deserves mentioning here. First of all, you can't ignore and not emphasize its resemblance to *Brahmagupta's formula*, which will be studied in this chapter. Secondly, the introduction of this classic formula and its proof will help motivate the development of the formulas for the area of quadrilaterals and the ideas behind the proofs to follow.

Heron's original proof made use of the properties of cyclic quadrilaterals, one of which we will consider below (we'll also use it as a very powerful tool in some problems examined in the later chapters):

A quadrilateral can be inscribed in a circle if and only if a pair of opposite angles is supplementary.



For proof of the direct statement let us recall that the measure of an inscribed angle in a circle is half the measure of the corresponding central angle:

$$\angle ABC = \frac{1}{2}\angle AOC, \text{ or } 2\angle ABC = \angle AOC. \qquad (*)$$

Draw the diameter DM. AO = DO = OM as radii of the same circle. Therefore, the triangle AOD is isosceles and $2\angle ADO = 180° - \angle AOD$, since $\angle ADO = \angle DAO$. Analogously in the isosceles triangle DOC, $2\angle CDO = 180° - \angle DOC$, since $\angle CDO = \angle DCO$.

By adding the equalities we obtain

$2\angle ADO + 2\angle CDO = 360° - \angle AOD - \angle DOC$, or

$2(\angle ADO + \angle CDO) = 360° - (\angle AOD + \angle DOC)$, which leads to $2\angle ADC = 360° - \angle AOC$. After substituting angle AOC from (*) and dividing both sides by 2, we get the final result:

$$\angle ADC = 180° - \angle ABC, \text{ or } \angle ADC + \angle ABC = 180°, \text{ or in words,}$$

If a quadrilateral is inscribed in a circle, its opposite angles are supplementary.

Let's now consider a quadrilateral ABCD whose opposite angles are supplementary: $\angle A + \angle C = \angle D + \angle B$. We have to prove that ABCD is a cyclic quadrilateral.

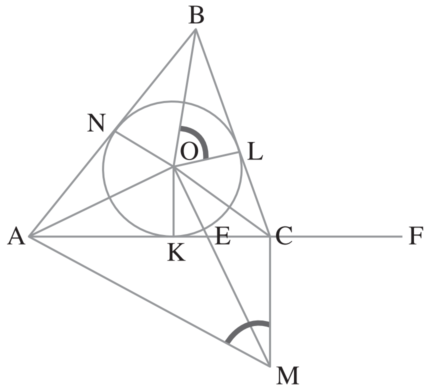
Let's draw a circumcircle around the triangle ABC and assume that it intersects AD at some point D′ different from D. The quadrilateral ABCD′ is cyclic and the pairs of its opposite angles are supplementary: $\angle A + \angle C = \angle D' + \angle B$.

By transitivity, $\angle B + \angle D' = \angle B + \angle D$, which implies $\angle D' = \angle D$. That is the contradiction to the assumption that points D and D′ do not coincide, which proves the fact that

If the opposite angles of a quadrilateral are supplementary, then it is a cyclic quadrilateral.

The above property allows us to arrive at an interesting proof of Heron's Formula.

In the triangle ABC let AB = c, BC = a, and AC = b, and $p = \frac{1}{2}(a + b + c)$. Denote by O the center of the incircle of triangle ABC. The perpendiculars dropped from O to the sides of ABC are equal as radii of the same circle: ON = OL = OK.



Recalling that O is the point of intersection of the angle bisectors of the triangle ABC, it's easy to prove the congruence of the pairs of right triangles OKA and ONA, ONB and OLB, and OLC and OKC. The pairs of corresponding sides in those triangles must be equal: AK = AN, NB = BL, and LC = CK. If we now locate point F on the extension of AC such that CF = BL, then the length of AF is equal to the semiperimeter of the triangle ABC. Indeed,

$$\begin{aligned} p &= \frac{1}{2}(\mathrm{AB} + \mathrm{BC} + \mathrm{CA}) = \frac{1}{2}(\mathrm{AN} + \mathrm{NB} + \mathrm{BL} + \mathrm{LC} + \mathrm{CK} + \mathrm{KA}) \\ &= \frac{1}{2}(2\mathrm{AK} + 2\mathrm{KC} + 2\mathrm{BL}) = \mathrm{AK} + \mathrm{KC} + \mathrm{CF} = \mathrm{AF}. \end{aligned}$$

In Chapter 2 it was proved that the area of a triangle is $S = \frac{1}{2}Pr$, where P is the triangle's perimeter and r is its inradius. Therefore, the area of triangle ABC equals $S = pr = \mathrm{OK} \cdot \mathrm{AF}$ (*).

The next step is to construct a perpendicular to AO through O and a perpendicular to AF through C. Denote the point of their intersection by M. The right triangles AOM and ACM share the common hypotenuse AM. Hence both triangles are inscribed in a common circle with AM as its diameter. It follows then that the quadrilateral AOCM is cyclic and so angles AOC and AMC are supplementary. Noting that the sum of pairs of equal angles at point O is 360°, it's not hard to see that

$$\angle AOB + \angle BOC + \angle COA = 2\angle AOK + 2\angle BOL + 2\angle COK = 360^\circ \text{ and}$$
$$\angle AOK + \angle BOL + \angle COK = 180^\circ.$$

But ∠AOK + ∠COK = ∠AOC, therefore ∠AOC + ∠BOL = 180°, which leads to a conclusion that ∠BOL = ∠AMC as supplementary to the same angle AOC (recall that angles AOC and AMC are supplementary in the cyclic quadrilateral AOCM). In the right triangles ACM and BLO two respective angles are equal, therefore they are similar. If we denote by E the point of intersection of OM and AC, then the right triangles OKE and MCE are similar as well (∠OKE = ∠MCE = 90° and ∠OEK = ∠MEC as vertical angles).

From the pairs of similar triangles we obtain that the ratios of the respective sides are equal: AC/BL = MC/OL and MC/OK = CE/EK. Recalling that OL = OK = r and dividing one equality by the other, we get that AC/BL = CE/EK. Substituting BL for CF (the segments are of the equal length by construction), we conclude that AC/CF = CE/EK.

Let's do a few simple algebraic manipulations with the above equality:

AC/CF + 1 = CE/EK + 1, so (AC + CF)/CF = (CE + EK)/EK, which implies that AF/CF = CK/EK. By multiplying the numerator and denominator by the same number AF on the right side and by AK on the left side, we get

$$\mathrm{AF}^2/(\mathrm{CF}\cdot\mathrm{AF}) = (\mathrm{CK}\cdot\mathrm{AK})/(\mathrm{EK}\cdot\mathrm{AK}). \tag{1}$$

Let's now consider the right triangle AOE (AO and OE are perpendicular by construction). By the geometric mean:

$$\mathrm{AK}\cdot\mathrm{EK} = \mathrm{OK}^2. \tag{2}$$

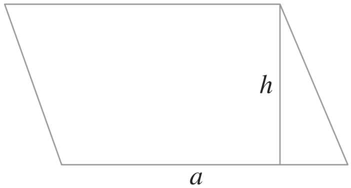
By substituting (2) into (1), we get that $\mathrm{AF}^2/(\mathrm{CF}\cdot\mathrm{AF}) = (\mathrm{CK}\cdot\mathrm{AK})/\mathrm{OK}^2$, from which $\mathrm{AF}^2\cdot\mathrm{OK}^2 = \mathrm{CF}\cdot\mathrm{AF}\cdot\mathrm{CK}\cdot\mathrm{AK}$ and $\mathrm{AF}\cdot\mathrm{OK} = \sqrt{\mathrm{CF}\cdot\mathrm{AF}\cdot\mathrm{CK}\cdot\mathrm{AK}}$. From (*) we conclude that the area of triangle ABC is $S = \sqrt{\mathrm{CF}\cdot\mathrm{AF}\cdot\mathrm{CK}\cdot\mathrm{AK}}$.

We showed before that AF = p. Obviously CF = AF – AC = $p - b$, CK = AF – (AK + CF) = $p - c$, and AK = AF – (KC + CF) = $p - a$.

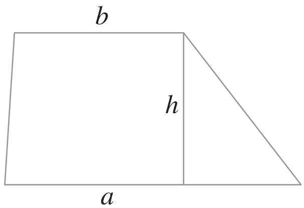
Therefore, after the substitution, the area of triangle ABC is $S = \sqrt{p(p-a)(p-b)(p-c)}$ and we are done.

At this point we are ready to move on from triangles to quadrilaterals.

While the calculation of the area of a triangle is presented in high school in a number of ways, as we mentioned above, for convex quadrilaterals there are area formulas only for parallelograms and trapezoids.



Area of a parallelogram $S = ah$ (a is the base, and h is the altitude dropped to it)



Area of a trapezoid $S = \frac{1}{2}(a+b)h$ (a and b are the bases and h is the altitude)

What about the general formula for the area of any convex quadrilateral? Wouldn't it be great to have it in your arsenal as a useful tool to solve area problems? Such a formula first was discovered by German mathematician Carl Anton Bretschneider (1808–1878) in 1842. It's interesting to note that another German mathematician, Karl Georg Christian von Staudt (1798–1867) got the same result independently in the same year.

In this chapter we will introduce Bretschneider's formula along with a few corollaries and demonstrate their practical application in problem solving. The techniques (proposed formulas) are interesting and timesaving, and can also be useful in giving a feel for the estimation of answers.

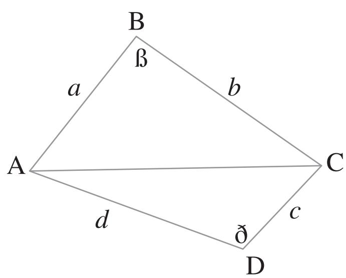
We will use some trigonometry to help us develop a formula in the main theorem below. We assume readers' familiarity with a few trigonometric formulas that will be used in the proof, such as $\cos^2 x + \sin^2 x = 1$, $\cos(x + y) = \cos x \cos y - \sin x \sin y$, and $1 + \cos x = 2\cos^2\frac{x}{2}$.

Theorem 1. *The area of a convex quadrilateral is*

$$S = \sqrt{(p-a)(p-b)(p-c)(p-d) - abcd \cdot \cos^2\left(\frac{ß + ð}{2}\right)},$$

where a, b, c, d are the sides of the quadrilateral ABCD, $ß = \angle ABC$, $ð = \angle ADC$, *and* $p = \frac{a+b+c+d}{2}$.

In all the next theorems and problems we will denote by *a, b, c,* and *d* the sides and by *p* the semiperimeter of a quadrilateral, and by ß and ð its opposite angles (unless it is specifically indicated otherwise).



Proof. Let us consider two triangles ABC and ADC. By the Law of Cosines
$AC^2 = a^2 + b^2 - 2ab \cdot \cos ß$ (from triangle ABC);
$AC^2 = c^2 + d^2 - 2cd \cdot \cos ð$ (from triangle ADC).
Then,

$$a^2 + b^2 - 2ab \cdot \cos ß = c^2 + d^2 - 2cd \cdot \cos ð, \text{ or}$$
$$a^2 + b^2 - c^2 - d^2 = 2ab \cdot \cos ß - 2cd \cdot \cos ð \qquad (1)$$

The area of ABCD is equal to the sum of the areas of triangles ABC and ADC:
$S = \frac{1}{2} \cdot ab \cdot \sin ß + \frac{1}{2} \cdot cd \cdot \sin ð$. Hence

$$4S = 2ab \cdot \sin ß + 2cd \cdot \sin ð. \qquad (2)$$

Square (1) and (2) and add:

$$(a^2 + b^2 - c^2 - d^2)^2 + 16S^2 = (2ab \cdot \cos ß - 2cd \cdot \cos ð)^2 + (2ab \cdot \sin ß + 2cd \cdot \sin ð)^2.$$
$$(a^2 + b^2 - c^2 - d^2)^2 + 16S^2 = 4a^2b^2 (\cos^2 ß + \sin^2 ß) + 4c^2d^2 (\cos^2 ð + \sin^2 ð)$$
$$- 8abcd(\cos ß \cos ð - \sin ß \sin ð),$$
$$(a^2 + b^2 - c^2 - d^2)^2 + 16S^2 = 4a^2b^2 + 4c^2d^2 - 8abcd \cdot \cos(ß + ð),$$
$$16S^2 = (4a^2b^2 + 4c^2d^2 + 8abcd) - 8abcd - 8abcd \cdot \cos(ß + ð) - (a^2 + b^2 - c^2 - d^2)^2.$$

Modifying the right-hand side we get

$$(2ab + 2cd)^2 - 8abcd - 8abcd \cdot \cos(ß + ð) - (a^2 + b^2 - c^2 - d^2)^2$$
$$= (2ab + 2cd)^2 - (a^2 + b^2 - c^2 - d^2)^2 - 8abcd(1 + \cos(ß + ð))$$
$$= (2ab + 2cd - a^2 - b^2 + c^2 + d^2)(2ab + 2cd + a^2 + b^2 - c^2 - d^2)$$
$$- 16abcd \cdot \cos^2\left(\frac{ß + ð}{2}\right)$$
$$= ((c + d)^2 - (a - b)^2)((a + b)^2 - (c - d))^2 - 16abcd \cdot \cos^2\left(\frac{ß + ð}{2}\right)$$
$$= (c + d + b - a)(c + d + a - b)(a + b + d - c)(a + b + c - d)$$
$$- 16abcd \cdot \cos^2\left(\frac{ß + ð}{2}\right).$$

After simplifying and noting that $2(p - a) = 2p - 2a = 2(a + b + c + d)/2 - 2a = a + b + c + d - 2a = b + c + d - a$ (the analogous equality holds for each side of the quadrilateral), we can rewrite the right-hand side of the last equality as

$$2(p - a)2(p - b)2(p - c)2(p - d) - 16abcd \cdot \cos^2\left(\frac{ß + ð}{2}\right).$$

Therefore,

$$16S^2 = 2(p - a)2(p - b)2(p - c)2(p - d) - 16abcd \cdot \cos^2\left(\frac{ß + ð}{2}\right).$$

Dividing both sides by 16, we get that

$S = \sqrt{(p - a)(p - b)(p - c)(p - d) - abcd \cdot \cos^2\left(\frac{ß + ð}{2}\right)}$, which was to be proved.

This theorem has three important corollaries.

The first one bears the name of the Indian mathematician Brahmagupta (598 CE–670 CE), whose contributions to the field of mathematics were quite substantial. He was the first to introduce arithmetic operations with 0, give general solutions to the linear and quadratic equations, and perform operations with fractions and negative numbers, to name just a few. In geometry his name is associated with the formula for the calculation of the area of cyclic quadrilaterals. It is amazing that in his works he did not provide any proof or even a hint as to how the formula was derived, so it is not clear how he got the result. In our exploration of his formula, the proof will be based on the results of the general formula from Theorem 1.

Theorem 2. (Brahmagupta's Formula)
If a quadrilateral is cyclic, then its area is

$$S=\sqrt{(p-a)(p-b)(p-c)(p-d)}.$$

Proof. As we proved at the beginning of the chapter, in a cyclic quadrilateral opposite angles are supplementary.

So, we see that ß + ð = 180° and therefore $\cos^2\left(\frac{ß+ð}{2}\right)=0$.

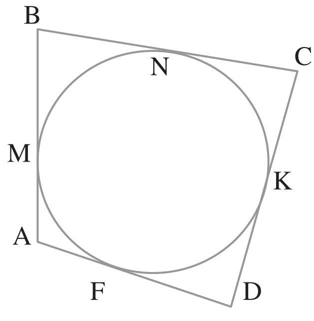
By Theorem 1, we obtain

$$S=\sqrt{(p-a)(p-b)(p-c)(p-d)}.$$

Theorem 3. *If a quadrilateral is circumscribed around a circle, its area is*

$$S=\sqrt{abcd\cdot\sin^2\left(\frac{ß+ð}{2}\right)}.$$

Proof. We begin by proving that in any circumscribed quadrilateral (tangential quadrilateral) two sums of the pairs of opposite sides are equal. This is known as *Pitot's Theorem*, named after the French engineer Henri Pitot.



For the proof it will suffice to recall that two tangent line segments from a point outside a circle have equal lengths:

AM = AF, BM = BN, CK = CN, and DK = DF. Adding we obtain:

AM + BM + CK + DK = AF + BN + CN + DF, or (AM + BM) + (CK + DK) = (AF + DF) + (BN + CN), which yields AB + CD = AD + BC.

Hence $a + c = b + d$.

Recalling that the semiperimeter $p = (a + b + c + d)/2$, and using this equality, it's easy to obtain that $p - a = c, p - b = d, p - c = a$, and $p - d = b$.

By Theorem 1, we get the desired formula:

$$S = \sqrt{abcd - abcd \cdot \cos^2\left(\frac{ß + ð}{2}\right)} = \sqrt{abcd \cdot \left(1 - \cos^2\left(\frac{ß + ð}{2}\right)\right)}$$
$$= \sqrt{abcd \cdot \sin^2\left(\frac{ß + ð}{2}\right)}.$$

Theorem 4. *If a quadrilateral is inscribed in a circle and is circumscribed around the circle simultaneously, its area is the square root of the product of its sides:*

$$S = \sqrt{abcd}.$$

Proof. Armed with Theorems 1 through 3, we don't even need to display the picture to solve this problem. By Theorem 2,

$S = \sqrt{(p-a)(p-b)(p-c)(p-d)}$ and we know that $a + c = b + d$ (the sum of the opposite sides of the circumscribed quadrilateral).

Therefore,

$$S = \sqrt{\frac{1}{2}(c + d + b - a) \cdot \frac{1}{2}(c + d + a - b) \cdot \frac{1}{2}(a + b + d - c) \cdot \frac{1}{2}(a + b + c - d)}$$
$$= \sqrt{(2/2)c \cdot (2/2)d \cdot (2/2)a \cdot (2/2)b} = \sqrt{abcd}.$$

It is interesting to note that Heron's formula for the area of a triangle may be derived as a corollary from Brahmagupta's formula. If $d = 0$, the cyclic quadrilateral becomes a triangle and then its area will be

$$S = \sqrt{p(p-a)(p-b)(p-c)}.$$

Usually Heron's formula is applied when the only information that is given about a triangle is the lengths of its sides. What about the formulas in this chapter? When would you make use of them?

I can't give you a certain answer, but most likely the formulas would be helpful in approaching problems where a quadrilateral has a given set of sides or opposite angles. You might get unexpected elegant and beautiful solutions simply by applying them. Let's look at some examples.

Problem 1. Of all quadrilaterals inscribed in a circle, find the one with the greatest area.

Solution. If a quadrilateral is inscribed in a circle and p is its semi-perimeter, then its area

$$\begin{aligned} S &= \sqrt{p(p-a)(p-b)(p-c)} = \sqrt{(p-a)(p-b)} \cdot \sqrt{(p-c)(p-d)} \\ &\le \frac{1}{2}((p-a)+(p-b)) \cdot \frac{1}{2}((p-c)+(p-d)) \\ &= \frac{1}{4}(a+b)(c+d) \le \frac{1}{16}(a+b+c+d)^2 = \left(\frac{P}{4}\right)^2, \end{aligned}$$

where P is the perimeter.

In the above manipulations we used the property that the arithmetic average of two positive integers is always greater or equal to their geometric average.

Note now that $S = \left(\frac{P}{4}\right)^2$ only when $a = b = c = d$. For a quadrilateral inscribed in a circle, that's possible only when it is a square. So, of all cyclic quadrilaterals, a square will have the greatest area.

It should be fairly easy for readers to prove another interesting statement, which follows directly from the above problem:

Of all quadrilaterals of a given perimeter, a square has the greatest area.

Problem 2. In a convex quadrilateral ABCD the lengths of the sides are 2, 2, 4, and 6. Its area is $5\sqrt{3}$. Prove that there exists a circle circumscribed around ABCD.

Solution. By the general formula from Theorem 1,

$$S = \sqrt{(p-a)(p-b)(p-c)(p-d) - abcd \cdot \cos^2\left(\frac{ß + ð}{2}\right)}.$$

Let's find p: $p = (2+2+4+6)/2 = 7$. After substituting all the values into the formula for the area of a quadrilateral, we'll get that

$$5\sqrt{3} = \sqrt{5 \cdot 5 \cdot 3 \cdot 1 - 2 \cdot 2 \cdot 4 \cdot 6\cos^2\left(\frac{ß + ð}{2}\right)}.$$

$$5\sqrt{3} = \sqrt{75 - 96\cos^2\left(\frac{ß + ð}{2}\right)}.$$

Squaring both sides leads to $75 = 75 - 96\cos^2\left(\frac{ß + ð}{2}\right)$, from which $96\cos^2\left(\frac{ß + ð}{2}\right) = 0$.

It follows that $\cos^2\left(\frac{ß+ð}{2}\right)=0$.

Therefore, $\frac{ß+ð}{2}=90°$ and $ß + ð = 180°$. So, the opposite angles of the quadrilateral are supplementary, which means that there exists a circle circumscribed around it.

Problem 3. Find the area of a convex quadrilateral with sides 2, 3, 3, 4 and opposite pair of angles 63° and 117°.

Solution. The sum of the opposite angles is 63° + 117° = 180°. Thus we have a cyclic quadrilateral. Its semiperimeter $p=\frac{1}{2}(2+3+3+4)=6$ and $p-2=4$, $p-3=3$, $p-4=2$.

Then by the formula from Theorem 2 we have $S=\sqrt{4\cdot 3\cdot 3\cdot 2}=6\sqrt{2}$.

Problem 4. ABCD is a tangential quadrilateral. The sum of the opposite angles of ABCD is 120°. Its area is $\frac{\sqrt{3}}{2}$.

Prove that the product of its sides is 1.

Solution. If ABCD is a quadrilateral circumscribed around a circle, then its area

$$S=\sqrt{abcd\cdot\sin^2\left(\frac{ß+ð}{2}\right)}=\sqrt{abcd\cdot\sin^2\left(\frac{120°}{2}\right)}=\sqrt{\frac{3}{4}abcd}.$$

Because $S=\frac{\sqrt{3}}{2}$,

$\frac{\sqrt{3}}{2}=\sqrt{\frac{3}{4}abcd}$, or after squaring both sides

$\frac{3}{4}=\frac{3}{4}abcd$, or $abcd=1$.

Problem 5. What is the maximum area of a convex quadrilateral with sides 1, 4, 7, and 8?

Solution. There are many different quadrilaterals with the given sides. They will differ from each other by the angles between sides. The area of any such quadrilateral may be calculated by the general formula

$$S=\sqrt{(p-a)(p-b)(p-c)(p-d)-abcd\cdot\cos^2\left(\frac{ß+ð}{2}\right)}.$$

Obviously, $S\le\sqrt{(p-a)(p-b)(p-c)(p-d)}$, with equality when $\cos^2\left(\frac{ß+ð}{2}\right)=0$. That occurs only when $ß+ð=180°$, which means that the maximum area will be found for a quadrilateral inscribed in a circle.

Let's find the value of p:

$p=\frac{1}{2}(1+4+7+8)=10$. Substituting into the formula for the area, we get $S=\sqrt{9\cdot 6\cdot 3\cdot 2}=18$.

Problem 6. The lengths of the sides of a cyclic quadrilateral form an arithmetic sequence with the common difference 2. Its area is $\sqrt{105}$. Find the length of each side.

Solution. Denote the length of the smallest side of the quadrilateral by $a_1 = x$. Then its sides will have the lengths, in ascending order, (by the definition of arithmetic sequence)

$$a_2 = x + 2$$
$$a_3 = x + 4$$
$$a_4 = x + 6.$$

The semiperimeter $p = \frac{1}{2}(x + (x+2) + (x+4) + (x+6)) = \frac{1}{2}(4x+12) = 2x+6$. Then

$$p-a_1 = x+6,\ p-a_2 = x+4,\ p-a_3 = x+2,\ p-a_4 = x.$$

The area of the cyclic quadrilateral is

$$S = \sqrt{(p-a_1)(p-a_2)(p-a_3)(p-a_4)}.$$

After substituting the value of S and the values of the factors, we get

$$\sqrt{105} = \sqrt{(x+6)(x+4)(x+2)x},$$
$$(x+6)(x+4)(x+2)x = 105,$$
$$((x+6)x)((x+4)(x+2)) = 105,$$
$$(x^2+6x)(x^2+6x+8) = 105.$$

To solve this equation we will introduce another variable:

$$y = x^2 + 6x, \tag{1}$$

which will transform the original equation into $y(y+8) = 105$, or $y^2 + 8y - 105 = 0$. This quadratic equation has two solutions: $y = -15$ or $y = 7$. Substituting y into (1), we get two equations,

$$x^2 + 6x + 15 = 0 \text{ and } x^2 + 6x - 7 = 0.$$

The first equation has no solutions.

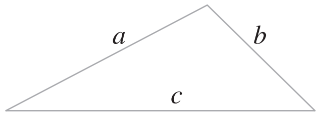
There are two solutions of the second equation: $x = 1$ or $x = -7$. Only the positive number satisfies the conditions of the problem.

Therefore $a_1 = 1, a_2 = x + 2 = 1 + 2 = 3, a_3 = x + 4 = 1 + 4 = 5,$ and $a_4 = x + 6 = 1 + 6 = 7$.

You'll certainly enjoy investigating different solutions to the above problems. I believe you will then truly appreciate the formulas for their efficiency and effectiveness.

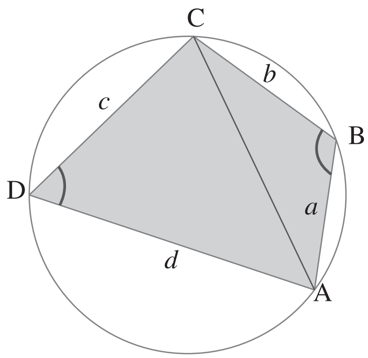
Heron' formula:

$$S=\sqrt{p(p-a)(p-b)(p-c)},\quad p=\frac{a+b+c}{2}$$



Brahmagupta's formula:

$$S=\sqrt{(p-a)(p-b)(p-c)(p-d)}$$



$$p=\frac{a+b+c+d}{2}\qquad \angle B+\angle D=180^\circ$$

MATHEMATICS

GEOMETRICAL KALEIDOSCOPE

BORIS PRITSKER

A high school course in geometry and some curiosity and enthusiasm for the subject are the only prerequisites for tackling this original exploration into the field, long a favorite for readers whose interest in math is not only practical and educational, but also recreational. The book centers on geometric thinking—what it means, how to develop it, and how to recognize it. Readers will discover fascinating insights into many aspects of geometry and geometrical properties and theorems, including such classic examples as Archimedes' law of the lever, Euler's line and circle properties, Fagnano's problem, and Napoleon's theorem.

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